Pair correlations and equidistribution

Christoph Aistleitner
TU Graz

Let $(x_n)_{n\geq 1}$ be a real sequence in the unit interval. Two possible tests for pseudorandomness of this sequence are to check whether it is equidistributed in the limit, and to check whether the distribution of its pair correlations agrees asymptotically with that of a randomly generated sequence (whose distribution of pair correlations is called "Poissonian").

Questions concerning equidistribution have played an important role in number theory since Hermann Weyl's seminal paper of 1916, while questions concerning pair correlations have been studied intensively in the last decades since they are connected with the distribution of quantum energy levels in the context of the Berry–Tabor conjecture. Roughly speaking, equidistribution considers the distributional properties of a sequence on a "global" scale, while pair correlations consider them on a "local" scale. It was widely understood that having Poissonian pair correlations is a "finer" property than equidistribution, but this was never made precise.

In our talk we prove that there actually is a strict implication: a sequence which has Poissonian pair correlations must be equidistributed as well. Beyond the proof of this result, we will also discuss other recent developments in this context, in particular relating the pair correlation statistic of certain parametric sequences with the concept of additive energy from additive combinatorics. The talk is based on joint work with Thomas Lachmann, Gerhard Larcher, Mark Lewko, and Florian Pausinger.