Counting points on Dwork hypersurfaces and hypergeometric functions

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N. Koblitz developed a formula for the number of points on diagonal hypersurfaces in the Dwork family in terms of Gauss sums. H. Goodson specializes Koblitz's formula to the family of Dwork K3 surfaces. She gives an expression for the number of points on the family of Dwork K3 surfaces $X_\lambda^4 : x_1^4 + x_2^4 + x_3^4 + x_4^4 = 4\lambda x_1x_2x_3x_4$ in the projective plane $\mathbb{P}^3(\mathbb{F}_q)$ over a finite field $\mathbb{F}_q$ in terms of Greene's finite field hypergeometric functions under the condition that $q \equiv 1 \pmod 4$. She then considers the higher dimensional Dwork hypersurfaces

$$X_\lambda^d : x_1^d + x_2^d + \cdots + x_d^d = d\lambda x_1x_2\cdots x_d.$$

She gives a formula for the number of points on X_λ^d in terms of finite field hypergeometric series and Gauss sums when $q \equiv 1 \pmod d$.

In this talk, we express the number of points on the Dwork hypersurface X_λ^d over a finite field of order $q \not\equiv 1 \pmod d$ in terms of McCarthy's p-adic hypergeometric function for any odd positive integer d which gives a solution to a conjecture of H. Goodson.