Effective results for division points on curves over finitely generated domains

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Let $A := \mathbb{Z}[z_1, \ldots, z_r] \supset \mathbb{Z}$ be a finitely generated integral domain over $\mathbb{Z}$ and denote by K the quotient field of A. Finiteness results for several kinds of Diophantine equations over A date back to the middle of the last century. S. Lang generalized several earlier results on Diophantine equations over the integers to results over A, including results concerning unit equations, Thue-equations and integral points on curves. However, all his results were ineffective.

The first effective results for Diophantine equations over finitely generated domains were published in the 1980's, when Győry developed his new effective specialization method. This enabled him to prove effective results over finitely generated domains of a special type.

In 2013 Evertse and Győry refined the method of Győry such that they were able to prove effective results for unit equations $ax+by = 1$ in $x, y \in A^*$ over arbitrary finitely generated domains A of characteristic 0. Later Bérczes, Evertse and Győry obtained effective results for Thue equations, hyper- and superelliptic equations and for the Schinzel-Tijdeman equation over arbitrary finitely generated domains.

In this talk I will present my effective results for equations $F(x, y) = 0$ in $x, y \in A^*$ for arbitrary finitely generated domains A, and for $F(x, y) = 0$ in $x, y \in \overline{\Gamma}$, where $F(X, Y)$ is a bivariate polynomial over A and $\overline{\Gamma}$ is the division group of a finitely generated subgroup Γ of K^*. These are the first effective versions over general finitely generated domains of the famous corresponding ineffective results of Lang (1960) and Liardet (1974).