Elliptic integrals of the third kind and 1-motives

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This is a joint work with Michel Waldschmidt. The generalized Grothendieck's conjecture of periods due to Y. André predicts that if M is a 1-motive defined over an algebraically closed subfield K of $\mathbb{C}$, then

$$\mathrm{tran.deg}_{\mathbb{Q}}\, K(\mathrm{periods}(M)) \geq \dim_{\mathbb{Q}} \mathcal{G}al_{\mathrm{mot}}(M_{\mathbb{C}})$$

where $K(\mathrm{periods}(M))$ is the field generated over K by the periods of M and $\mathcal{G}al_{\mathrm{mot}}(M_{\mathbb{C}})$ is the motivic Galois group of the 1-motive.

In her PhD thesis, the first author has investigated this conjecture in the case of 1-motives of the form $M = [u : \mathbb{Z}^r \longrightarrow \prod_{j=1}^{n} \mathcal{E}_j \times \mathbb{G}_m^s]$ and she has showed that the generalized Grothendieck's conjecture applied to these 1-motives is equivalent to a transcendental conjecture involving elliptic integrals of the first and second kind, and logarithms of complex numbers.

The aim of this paper is to investigate the generalized Grothendieck's conjecture for arbitrary 1-motives, starting with 1-motives of the form $M = [u : \mathbb{Z}^r \longrightarrow G]$, whose underlying extension G of an elliptic curve by a torus is not split. This will imply the introduction of elliptic integrals of the third kind in the computation of the period matrix and therefore the generalized Grothendieck's conjecture applied to those 1-motives will be equivalent to a transcendental conjecture involving elliptic integrals of the first, second and third kind.