Integral p-adic Hodge theory and THH

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Given a smooth proper variety of the ring of integers of a p-adic field, we recently showed that the mod p Betti numbers of the generic fibre are a lower bound for the de Rham Betti numbers of the special fibre. The main new innovation was the construction of an integral p-adic cohomology theory that gets rid of certain factorials occurring as denominators in crystalline cohomology.

In a different context, Waldhausen suggested many decades ago that changing the coefficient ring to the sphere spectrum instead of the integers might lead to better behaved answers in homotopy theory. An early piece of evidence was Bokstedt's work on the Hochschild homology of a finite a field: the theory over the integers gives a divided power polynomial ring, while the theory over the sphere spectrum gets rid of the divided powers and produces a genuine polynomial ring. Much more recently, Hesselholt observed a similar phenomenon for the valuation ring of an algebraically closed p-adic field.

In my talk, I'll give an overview of each of these stories, and then explain why they are closely linked: the p-adic cohomology theory mentioned in the first paragraph can be recovered as a suitable graded piece of (an enrichment of) the Hochschild homology of the variety relative to the sphere spectrum.

(This is a report on joint work with Matthew Morrow and Peter Scholze.)