Waring's problem with shifts

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In its original form, Waring's problem asks whether every positive integer can be written as the sum of s kth powers of natural numbers, where s depends only on k. In this talk, I will discuss an analogue of this problem in which we attempt to approximate a large, positive real number τ by a sum of "shifted" kth powers.

I will outline the Davenport–Heilbronn method, which allows us to obtain an asymptotic formula for the number of solutions to the relevant Diophantine inequality whenever $s \geq k^2+(3k-1)/4$, improving on the best previously known result. I will also show that there are arbitrarily large τ which cannot be approximated in this way if we insist on the kth powers being too close together.