Sparse generalised polynomials

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We investigate generalised polynomials (i.e. polynomial-like expressions involving the use of the floor function) which take the value 0 on all integers except for a set of density 0. By a theorem of Bergelson-Leibman, generalised polynomials can be completely described in terms of dynamics on nilmanifolds. Our main result is that the set of integers where a sparse generalised polynomial takes non-zero value cannot contain a translate of an IP set. We also study some explicit constructions, and show that the characteristic functions of the Fibonacci and Tribonacci numbers are given by generalised polynomials. Finally, we show that any sufficiently sparse $\{0,1\}$-valued sequence is given by a generalised polynomial. We apply these results to a question on automatic sequences. The talk is based on joint work with Jakub Konieczny.

[BK], Jakub Byszewski, Jakub Konieczny, *Sparse generalised polynomials*, arxiv.org/abs/1612.00073, to be published in Trans. AMS.