Graph theory and uniform finiteness results for rational points on curves

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The talk is about a joint work with Fumiharu Kato (Tokyo) and Janne Kool (Copenhagen) in which we describe a graph theoretical method to obtain uniform finiteness results for rational points on curves. More specifically, I will explain the following result from [CKK]: Let X denote a curve over a global function field K, such that its Jacobian does not admit a $\bar{K}$-morphism to a curve defined over a finite field. Let K_∞ denote the completion of K at a place ∞, and let G denote the stable reduction graph of X/K_∞. Let Δ denote the maximal vertex degree of G, $|G|$ the number of vertices of G and λ the smallest non-zero eigenvalue of the Laplacian of G. Then the set of rational points on X of degree at most $(\lambda(|G|-1)-4\Delta-4)/(2\lambda+8\Delta+8)$ over K is finite. The result is deduced from a lower bound for gonality of graphs (similar to the Li-Yau inequality in differential geometry) and can be applied to Drinfeld modular curves to deduce a lower bound for the degree of a modular parametrisation of an elliptic curve over a function field in terms of the conductor.

[CKK] Gunther Cornelissen, Fumiharu Kato & Janne Kool, “A combinatorial Li–Yau inequality and rational points on curves”, Math. Ann. 361, 211-258 (2015).