Dirichlet Series Associated To Recurrence Series

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Analytic continuation of Fibonacci zeta function, i.e

$$Fib(s) = \sum_{n=1}^{\infty} \frac{1}{F_n^s}, \quad \Re(s) > 0,$$

where F_n are Fibinacci numbers, was shown by the author [1], and L. Navas [2], independently.

In this talk we generalize their result to general recurrence series. Our results contains, for example, the following

Proposition. *Let T_n be the "Tribonacci" numbers, i.e. defined by*

$$T_{n+3} = T_{n+2} + T_{n+1} + T_n, \quad n > 0, \quad T_3 = T_2 = T_1 = 1.$$

The Dirichlet series defined by

$$Trib(s) = \sum_{n=1}^{\infty} \frac{1}{T_n^s}, \quad \Re(s) > 0.$$

can be continued meromorphically to the whole complex plane.

We also would like to discuss some relations to the works of Hecke and Hardy-Littlewood around 1920.

[1] S. Egami, Some curious Dirichlet series, RIMS Kokyuroku 1091 (1999), 172-174.

[2] L. Navas, Analytic continuation of the Fibonacci Dirichlet series. Fibonacci Q.39 (2001), 409-418.