On the local-global divisibility problem over GL_2-type varieties

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We are interested in a problem called the Local Global divisibility problem over Abelian Varieties. Let k be a number field and let A/k be an abelian variety of dimension d.

Problem. Let P be in $A(k)$ and let q be a positive integer. Suppose that for all but finitely many places v of k, there exists $D_v \in A(k_v)$ such that $P = qD_v$. Does there exist $D \in A(k)$ such that $P = qD$?

Dvornichich and Zannier gave a cohomological interpretation based on the Galois representations over torsion points. Using this interpretation, Laura Paladino, Gabriele Ranieri and Evelina Viada gave a criterion for the Local-global divisibility problem in the particular case of Elliptic Curves. But in higher dimension, the size of the representation makes the problem technically more difficult. For this reason we studied the special case of GL_2-type varieties. An abelian variety A/k is of GL_2-type if there exists a number field $E/\mathbb{Q}$ of degree equal to d, such that an order R of E embeds in the endomorphism ring of A. This allows us to associate a p-adic Galois representation with values in $\mathrm{GL}_2(R \otimes \mathbb{Z}_p)$, in the place of $\mathrm{GL}_{2d}(\mathbb{Z}_p)$. In a recent work with Gabriele Ranieri, we generalize the previous criterion known for Elliptic Curves in the case of GL_2-type abelian varieties. In our talk we explain this last result.