Diophantine Triples in Linear Recurrences

Christoph Hutle
University of Salzburg

One of the oldest problems in Number Theory is the question of Diophantus, which is about constructing sets of rationals or integers with the property that the product of any two of its distinct elements plus 1 is square.

Recently several variations of this problem have been investigated. The problem of finding bounds on the size m for Diophantine m-tuples with values in linear recurrences is one such variation.

Let k be a fixed integer ≥ 2. It was shown in 2015 by C. Fuchs, C. Hutle, F. Luca and L. Szalay, that for any k-generalized Fibonacci sequence $F^{(k)}$ given by $F_0^{(k)} = \ldots = F_{k-2}^{(k)} = 0, F_{k-1}^{(k)} = 1$ and

$$F_{n+k}^{(k)} = F_{n+k-1}^{(k)} + \cdots + F_n^{(k)}$$

for all $n \geq 0$, there are only finitely many Diophantine triples with values in $F^{(k)}$.

In this talk, it will be discussed, how this result can be generalized to a much larger class of linear recurrences. We will show that the existence of infinitely many Diophantine Triples can be explained by certain functional equations, which need to be satisfied.