Polynomial Pell equations $P(x)^2 - (x^{2m} + ax + b)\,Q(x)^2 = 1$ and hyperelliptic curves

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The title equations are connected with Jacobians of hyperelliptic curves $C_{m,a,b} : y^2 = x^{2m} + ax + b$ defined over $\mathbb{Q}$. More precisely, these equations have a nontrivial solution if and only if the class of the divisor $\infty^+ - \infty^-$ is a torsion point in Jacobian $Jac(C_{m,a,b})$, where ∞^+ and ∞^- are two points at infinity in $C_{m,a,b}$.

We show that if $ab = 0$ then the title equations have nontrivial solutions (and we write explicit formulas). On the other hand, we prove that for any $m > 1$ there exists infinitely many pairs (a, b) such that our equations have no nontrivial solutions. Moreover, for $m = 2, 3$ for almost all (a, b) with $ab \neq 0$, these equations have no nontrivial solutions. We also give infinitely many explicit examples when nontrivial solution do not exist.