

Continued fraction expansions of the quasi-arithmetic power means of two positive definite matrices

Ali KACHA
Ibn Tofail University

This work is done jointly with Badr Nejjar and Salah Salhi. Let A and B be two definite positive matrices, α be a real number such that $0 \leq \alpha \leq 1$ and p be a strictly positive integer. The quasi-arithmetic power mean with parameter (p, α) , is defined by :

$$f_{p,\alpha}(A, B) = A^{1/2}((1 - \alpha)I + \alpha(A^{-1/2}BA^{-1/2})^p)^{1/p}A^{1/2}$$

In a practical context, the computation of $f_{p,\alpha}(A, B)$ imposes many difficulties by virtue of the appearance of the rational exponents of matrices. One fundamental motivation and goal of this paper is to remove this difficulty and reveal a practical and efficient method involving matrix continued fractions, for the computation of $f_{p,\alpha}(A, B)$. Numerical applications are given to illustrate the importance of our approach. Here are some references of our work :

L. Lorentzen, H. Waadeland, "Continued fractions with applications", Elsevier Science Publishers, (1992).

M. Raïssouli, A. Kacha and S. Salhi, "Continued fraction expansion of real powers of positive definite matrices with applications to matrix means", Arab J Sci Eng, V. 31, N.1A, (2006), 41-55.