Fully maximal and minimal supersingular varieties

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We consider a supersingular abelian variety A defined over a finite field K; we say that A is maximal (resp. minimal) over K if all its normalised Weil numbers over K are -1 (resp. 1). The (normalised) Weil numbers of A determine A up to isogeny. We ask whether A and its K-twists (which may have different Weil numbers) become maximal over a finite extension of K, and classify this behaviour by defining three possible types for A: fully maximal, fully minimal, and mixed. We analyse these types for supersingular abelian varieties and curves, under restrictions on their automorphism group.

In particular, we give a complete characterisation of the type of all supersingular abelian varieties of dimension $g = 1, 2$ in arbitrary characteristic. This uses the classifications of isogeny classes due to Tate and Waterhouse ($g = 1$), and Maisner-Nart and Howe-Nart-Ritzenthaler ($g = 2$). Moreover, we obtain a similar characterisation for supersingular genus 3 curves in characteristic 2, using a parametrisation of the moduli space by Viana-Rodriguez, and performing a careful analysis on the automorphism groups. This is joint work with Rachel Pries.