On the equation $x + y = 1$ in finitely generated multiplicative groups in positive characteristic

Peter Koymans
Universiteit Leiden

This is a joint work with Carlo Pagano.

Let K be a field of characteristic $p > 0$ and let G be a subgroup of $K^* \times K^*$ with $\dim_{\mathbb{Q}}(G \otimes_{\mathbb{Z}} \mathbb{Q}) = r$ finite. Then Voloch proved that the equation $ax + by = 1$ in $(x, y) \in G$ for given $a, b \in K^*$ has at most $p^r(p^r + p - 2)/(p - 1)$ solutions $(x, y) \in G$, unless $(a, b)^n \in G$ for some $n \geq 1$.

Voloch also conjectured that this upper bound can be replaced by one depending only on r. Our main theorem answers this conjecture positively. We prove that there are at most $31 \cdot 19^{r+1}$ solutions (x, y) unless $(a, b)^n \in G$ for some $n \geq 1$ with $(n, p) = 1$.

During the proof of our main theorem we generalize the work of Beukers and Schlickewei to positive characteristic, which heavily relies on diophantine approximation methods. This is a surprising feat on its own, since usually these methods can not be transferred to positive characteristic.