Rational Curves on Cubic Hypersurfaces over $\mathbb{F}_q$.

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Using a version of the Hardy – Littlewood circle method over $\mathbb{F}_q(t)$, one can count $\mathbb{F}_q(t)$-points of bounded degree on a smooth cubic hypersurface $X \subset \mathbb{P}^{n-1}_{\mathbb{F}_q}$. Moreover, there is a correspondence between the number of $\mathbb{F}_q(t)$-points of bounded height and the number of $\mathbb{F}_q$-points on the moduli space $\mathrm{Mor}_d(\mathbb{P}^1_{\mathbb{F}_q}, X)$, which parametrises the rational maps of degree d on X. In this talk I will show that for $n \geq 10$, and q and d large enough, there exists a rational curve defined over $\mathbb{F}_q$ on X passing through two fixed points, one of which must not belong to the Hessian. Moreover, I will give an asymptotic formula for the number of such curves.