A local computation of polynomial sums over distinct elements of a group

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Given a noetherian commutative ring R, a finite group $G \subseteq R$ and a polynomial $f \in R[x_1, \ldots, x_n]$, we give a formula for

$$\sum_{\substack{x_1,\ldots,x_n \in G \\ \text{distinct}}} f(x_1, \ldots, x_n)$$

in terms of simpler sums, i.e. $\sum_{x \in H} x$, where H is a subgroup of G.

The computation of the latter sum is reduced to the case $H = \mathbb{Z}/p^k\mathbb{Z}$ and $(R, \mathfrak{m})$ local artinian ring. In some special rings this sum happens to be always 0 or $|H|$, regardless of H. We recover a formula in the general case for "good rings", which in particular includes $R = \mathbb{Z}/n_1\mathbb{Z} \times \ldots \times \mathbb{Z}/n_k\mathbb{Z}$.