On the distribution of the Gaussian primes

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This is a joint work with Masanori Hagihara (Kosei-Gakuen High School). For a natural number n, let q_n be the n-th prime number satisfying $q_n \equiv 1 \pmod 4$. Then we have a unique prime element π_n of the ring of Gaussian integers (i.e. π_n is a Gaussian prime) which satisfy the conditions

$$|\pi_n|^2 = q_n, \quad \pi_n \equiv 1 \pmod{2+2i}, \quad \operatorname{Im} \pi_n > 0,$$

where $i = \sqrt{-1}$, $|z|$ is the absolute value of z, and Im means the imaginary part. Our subject of study is the distribution of π_n for $n \geq 1$. Namely, we computed π_n for many n by using Maple, and examined their distribution numerically.

We first consider the argument of π_n, i.e. $\theta_n = \arg \pi_n (0 < \theta_n < \pi)$. It is proved by Hecke that θ_n is uniformly distributed in the interval $(0, \pi)$. But, our computation indicates that the difference $\theta_{n+1} - \theta_n$ is NOT uniformly distributed. In the talk, we present some graphs which shows the behavior of the sequence $\theta_{n+1} - \theta_n$.

Our second concern is the distance between π_n's. In particular, we will show our data which suggest that the equality $|\pi_m - \pi_n|^2 = 8$ holds for infinitely many pairs of m and n. (Note that, by the definition of π_n, $|\pi_m - \pi_n|^2$ is a multiple of $8 = |2 + 2i|^2$.)