Classical weight one forms in a Hida family of Hilbert cusp forms with parallel weights

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This is my recent work appeared in [O]. Let F be a totally real field and p an odd prime. We consider a primitive p-ordinary Λ-adic cusp form $\mathcal{F}$ of parallel weight Hilbert cusp forms defined over F, whose notion was first invented by Wiles. It is proved by Hida that a specialization of $\mathcal{F}$ at any arithmetic point of weight two or more is a classical (holomorphic) Hilbert cusp form. However, this is not always the case for weight one specializations.

In the case of $F = \mathbb{Q}$, Ghate and Vatsal showed in [GV] that a primitive p-ordinary Λ-adic cusp form admits infinitely many classical weight one specializations if and only if it is a CM family (namely, constructed from a Hecke character of an imaginary quadratic field). Further the number of such forms inside a non-CM family is bounded by an explicit constant due to Dimitrov and Ghate in [DG]. The former result in [GV] was generalized to the case of totally real fields by Balasubramanyam, Ghate and Vatsal in [BGV], which contains an outline of a proof for nearly ordinary families with not necessarily parallel weights as well.

In this talk, I will describe how to give an explicit estimate on the number of classical weight one specializations obtained from a non-CM primitive p-ordinary Λ-adic cusp form of Hilbert cusp forms with parallel weights.

References

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