Polynomials and progression-free sets

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In this talk we will look at a new variant of the polynomial method which was first used to show that sets avoiding 3-term arithmetic progressions in groups like $\mathbb{Z}_4^n$ and $\mathbb{F}_q^n$ are exponentially small (compared to the size of the group). Namely, for $\mathbb{Z}_4^n$ with Croot and Lev we improved the bound $\frac{4^n}{n(\log n)^\varepsilon}$ to 3.62^n and for $\mathbb{Z}_3^n$ Ellenberg and Gijswijt the bound $\frac{3^n}{n^{1+\varepsilon}}$ to 2.756^n. Since then many interesting applications of this method were shown, for instance the solution of the Erdős-Szemerédi sunflower conjecture.