Cohen-Lenstra for ray class groups: Model and theorems

Carlo PAGANO Universiteit Leiden

I will present a joint work with Effhymios Sofos. In 1983 Cohen and Lenstra provided an heuristic model for the class group of quadratic number fields, viewed as an abelian group. In 2016 Ila Varma computed the average 3torsion of ray class group of fixed integral conductor for real and imaginary quadratic number fields. She asked if it was possible to interpret her results by means of a generalization of the Cohen-Lenstra model. In this talk I will explain the two main goals obtained in this paper:

Goal 1: We construct a generalization of the Cohen-Lenstra heuristics for ray class groups, viewed as short exact sequences of Galois modules, in the case of imaginary quadratic number fields. The consequent heuristic value of the average *p*-torsion of the ray class group agrees with the theorem of Varma for p = 3.

Goal 2: We prove that the heuristic constructed in Goal 1 is correct for a map arising in a long exact sequence obtained by taking 4-torsion, when the primes in the conductor of the ray are unramified in the imaginary quadratic number field. This gives even stronger evidence towards the above new conjecture. One obtains, as a cruder result, the joint distribution of the 4-rank of ray class group (of fixed integral conductor) and ordinary class group. The result and the methods generalize those introduced by Fouvry and Kluners for the ordinary class group.