Theorems of Barban-Davenport-Halberstam type with higher moments

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In an arithmetic progression $(a+nq)_{n\geq 1}$ we know that up to x there are around $x/\phi(q)$ primes. The error $E(x;q,a)$ in this approximation is expected to be not much bigger than $\sqrt{x/q}$, but general knowledge is nowhere near this. On average, however, we do know this, and this statement is the Barban-Davenport-Halberstam Theorem:

$$\sum_{\substack{1\leq a\leq q\leq Q\\(q,a)=1}} E(x;q,a)^2 \ll_A Qx\log x+\frac{x^2}{(\log x)^A}, \quad \text{for } Q\leq x \text{ and every } A>0;$$

and you could ask if a similar statement might be true for the higher moments. We will investigate the higher moments for some other sequences than the primes.