**Intersective $A_5$ polynomials with two irreducible factors**

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It is a joint work with Jack Sonn. A monic polynomial in one variable with rational integer coefficients is called *intersective* if it has a root modulo $m$ for all positive integers $m$, and *nontrivially intersective* if in addition it has no rational root, "intersective" will always mean "nontrivially intersective". Let $G$ be a finite noncyclic group and let $r(G)$ be the smallest number of irreducible factors of an intersective polynomial with Galois group $G$ over $\mathbb{Q}$. There is a group-theoretically defined lower bound for $r(G)$, given by the smallest number $s(G)$ of proper subgroups of $G$ having the property that the union of the conjugates of those subgroups is $G$ and their intersection is trivial. This follows from

**Proposition.** *Let $K/\mathbb{Q}$ be a finite Galois extension with (noncyclic) Galois group $G$. The following are equivalent:*

1. *$K$ is the splitting field of a product $f = g_1 \cdots g_m$ of $m$ irreducible polynomials of degree greater than 1 in $\mathbb{Q}[x]$ and $f$ has a root in $\mathbb{Q}_p$ for all (finite) primes $p$.*
2. *$G$ is the union of the conjugates of $m$ proper subgroups $H_1, ..., H_m$, the intersection of all these conjugates is trivial, and for all (finite) primes $\mathfrak{p}$ of $K$, the decomposition group $G(\mathfrak{p})$ is contained in a conjugate of some $H_i$.*

We will call an intersective polynomial $f(x)$ with Galois group $G$ over $\mathbb{Q}$ *optimally intersective for $G$* if the above lower bound $s(G)$ for $r(G)$ is attained, i.e. $f(x)$ is the product of $s(G)$ irreducible factors. Accordingly, a Galois extension $K/\mathbb{Q}$ with Galois group $G$ will be called an *optimally intersective realization of* $G$ if it is the splitting field of a polynomial which is optimally intersective for $G$. In fact, for all finite solvable groups there exist infinitely many disjoint optimally intersective realizations. Among the (noncyclic) alternating groups $A_n$, $s(A_n) = 2$ if and only if $n \in \{4, 5, 6, 7, 8\}$. In the case of $A_4$, there is a direct and explicit proof (showed by *Lee, Spearman and Yang*). We will prove the following theorem

**Theorem.** *There exist infinitely many disjoint optimally intersective realizations of $A_5$.*

which says that the same result holds for the non-solvable group $A_5$.