The Hasse Norm Principle for Biquadratic Extensions of $\mathbb{Q}$

Nick ROME
University of Bristol

In 1931, Hasse showed that for every cyclic extension L/K of number fields whenever an element of K^* is locally representable as the norm of an element, it must also be globally representable as a norm. This prorperty is known as the Hasse norm principle. Hasse showed that this need not hold for a general abelian extension.

We investigate the simplest possible non-cyclic abelian extension of $\mathbb{Q}$, the biquadratics with Galois group $G = (\mathbb{Z}/2\mathbb{Z})^2$, and develop asymptotics for how common it is for such extensions to fail the Hasse norm principle.