On absolutely normal numbers and their discrepancy estimate

Adrian Scheerer
TU Graz

A real number is called *normal to base* b, $b \geq 2$ an integer, if in its base-b expansion all finite blocks of digits occur with the expected asymptotic frequency. If a real number is normal to all integer bases $b \geq 2$ it is called *absolutely normal*. It is an old problem, going back to Borel, to exhibit an absolutely normal number. Recent progress on the construction of these numbers include a polynomial-time algorithm by Becher, Heiber and Slaman, who constructed an absolutely normal number by giving its digits to some base one after the other. Their algorithm sacrifices speed of convergence to normality in order to achieve polynomial-time complexity. It is unknown whether low time-complexity necessarily implies slow convergence to normality.

In this talk we present a recursive construction of the base-2 expansion of an absolutely normal number x, such that for any integer $b \geq 2$, the discrepancy of the sequence $(b^n x)_{n\geq 0}$ is essentially the same as realized by almost all real numbers. Our construction has triple exponential time-complexity, but improves a result by Levin in terms of discrepancy.

Joint work with Verónica Becher and Theodore Slaman.