Estimates for product and quotient sets of integers and rational numbers of bounded height

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In my talk I am going to present and discuss some new results about quotient and product sets of rational numbers and integers :

Proposition.— *Let* $A, B \subseteq F_Q$, *where* $F_Q = \{\frac{r}{s}, 1 \le r, s \le Q\}$. *Then there exists an absolute constant* $C > 0$, *such that then we have the following estimate*

$$|AB| \ge |A||B| exp\Big\{(-C + o(1))\frac{\log Q}{\log\log Q}\Big\}, Q \to \infty,$$

[DD] J. Bourgain, S. Konyagin and I. Shparlinski "Product sets of rationals, multiplicative translates of subgroups in residue rings and fixed points of the discrete logarithm", Int. Math. Res. Not., 090, 1-29 (2008).