The reconstruction of global fields from Dirichlet L-series and the abelianized Galois group

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Many invariants of a global field do not uniquely determine the field itself. One such example is the zeta function: there exist two number fields K, L of degree 7 such that $\zeta_K(s) = \zeta_L(s)$, but $K \not\simeq L$ ([Per], Section 4).

The famous *Neukirch-Uchida theorem* ([Neu], [Uch]) states that two global fields K and L are isomorphic precisely when their absolute Galois groups G_K and G_L are isomorphic. However, it is also known that an isomorphism of the *abelianized* Galois groups G_K^{ab} and G_L^{ab} does not provide sufficient information. In this talk we present two theorems which combine the abelianized Galois group with additional invariants of the field, so that they together determine the isomorphism type of the underlying global field.

The first theorem states that the combination of Dirichlet L-series and the abelianized Galois group of a global field determines the field uniquely. In the case of number fields, a stronger statement holds: every number field has an Dirichlet L-series that no other number field has.

Secondly, we introduce a topological monoid constructed from the integral ideles and the abelianized Galois group, isomorphic to the Deligne-Ribet monoid. When this topological monoid is equipped with a suitable action of the integral ideals of the field, it determines the underlying field.

[Per] Robert Perlis "On the equation $\zeta_K(s) = \zeta_{K'}(s)$", Journal of Number Theory, 9, 342-360 (1977).

[Neu] Jürgen Neukirch, "Kennzeichnung der p-adischen und der endlichen algebraischen Zahlkörper", Inventiones Mathematicae, 6, 296-314 (1969).

[Uch] Kôji Uchida, "Isomorphisms of Galois groups", J. Math. Soc. Japan 28, 4, 617-620 (1976).