The 2-adic valuation of some generalized Fibonacci sequences with an application to Diophantine equations involving factorials

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For a given integer $k \geq 2$ define a generalized Fibonacci sequence $\{t_k(n)\}_{n\geq 0}$ using the recurrence

$$t_k(n+k) = \sum_{i=0}^{k-1} t_k(n+i),$$

where $t_k(0), ..., t_k(k-1)$ are fixed.

A method of determining the 2-adic valuation of the terms $t_k(n)$ will be shown, which could also be generalized for other prime numbers p. In particular, I will derive an explicit formula for $\{\nu_2(t_k(n))\}_{n\geq 0}$ when k is even and the initial terms are $t_k(0) = 0, t_k(1) = \cdots = t_k(k-1) = 1$. A similar problem has already been considered by Lengyel and Marques [1,2] for $k \in \{3, 4, 5\}$.

The result will be applied to effectively solve Diophantine equations of the form

$$\prod_{j=1}^{d} t_k(n_j) = m!$$

with respect to $n_1, ..., n_d, m$, where $d \geq 1$ is a fixed integer. I will argue that the algorithm of solving the equation also works for a specific, more general family of sequences.

At the end, I will briefly discuss how the results are related to p-regular sequences, in particular, the work of Shu and Yao [3], who gave a criterion for p-regularity of the p-adic valuation of binary recurrence sequences. Most of the presented results can be found in my paper [4].

[1] T. Lengyel and D. Marques, The 2-adic order of the Tribonacci Numbers and the equation $T_n = m!$, *J. Integer Seq.* **17**: Article 14.10.1 (2014).

[2] T. Lengyel and D. Marques, The 2-adic order of some generalized Fibonacci numbers, *Integers: Electronic Journal of Combinatorial Number Theory* **17**: A5 (2017).

[3] Z. Shu and J.-Y. Yao, Analytic functions over $\mathbb{Z}_p$ and p-regular sequences, *C. R. Math.* **349**: 947–952 (2011).

[4] B. Sobolewski, *The 2-adic valuation of generalized Fibonacci sequences with an application to certain Diophantine equations*, preprint: arXiv:1702.05819v1 (2017).