

Artin's conjecture and the ternary Goldbach problem

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Fix an integer g which is neither a square nor -1 . Artin's conjecture states that g is a primitive root modulo infinitely many primes and provides a prediction for the density of such primes. Hooley [H] established the conjecture conditionally on the generalised Riemann hypothesis but there is still no single value of g for which it is known to hold unconditionally.

We establish, conditionally on GRH, an asymptotic for the number of representations of an odd integer as a sum of three primes, all three of which have g as a primitive root.

There is an extensive literature concerning solutions of Diophantine equations within infinite sets of primes; such work covers the case of sets of primes that admit a simple Chebotarev description or sets that are sufficiently dense in suitable arithmetic progressions of the integers. None of this covers our situation. Our approach is built upon a connection of Hooley's ideas and the Hardy–Littlewood circle method via the use of the linear sieve, while Lenstra's work [L] is used to provide a local to global statement regarding the existence of solutions. This is joint work with C. Frei and P. Koymans.

[H] Hooley C., "On Artin's conjecture", *J. reine angew. Math.*, 225, 209-220 (1967).

[L] Lenstra, Jr., H. W., "On Artin's conjecture and Euclid's algorithm in global fields", *Invent. Math.*, 42, 201-224 (1977).