On the solutions of a Diophantine equation with power sums

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This is joint work with Attila Bérczes, István Pink and Gamze Savaş. In this work, we are interested in positive integer solutions of the Diophantine equation

$$T_k(x) = y^n$$

where $T_k(x) = (x+1)^k + (x+2)^k + ... + (2x)^k$.

We first provide upper bounds for n which depend on assertions describing the precise exponents of 2 and 3 appearing in the prime factorization of $T_k(x)$ and on the explicit solution of polynomial exponential congruences. Secondly, we show that the equation has no solutions in positive integer unknowns (x, y, k, n) with $2 \leq x \leq 13$, $y \geq 2$, $k \geq 1$, $n \geq 3$. To prove this, we combine several tools: Baker's method (in particular, sharp bounds for the linear combinations of logarithms of two algebraic numbers), polynomial-exponential congruences and computational methods.