Digits in finite fields

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The study of the connection between the arithmetic properties of an integer and the properties of its digits in a given basis produces a lot of interesting questions and a lot of papers have been devoted to this topic. In the context of finite fields, the algebraic structure permits to formulate and study new problems of interest which might be out of reach in $\mathbb{N}$. Dartyge and Sárközy [2] initiated the study of the concept of digits in $\mathbb{F}_q$, establishing estimates for the number of squares whose sum of digits is fixed. They also obtained results for polynomial values, resp. polynomial values with primitive element arguments whose sum of digits is fixed.

We will study new questions in this spirit: 1) give (more precise) estimates for the number of elements of $\mathbb{F}_q$ that belong to a special sequence and whose sum of digits is fixed; 2) given subsets $\mathcal{C}$ and $\mathcal{D}$ of $\mathbb{F}_q$, find conditions on $|\mathcal{C}|$ and $|\mathcal{D}|$ to ensure that there exists $(c, d) \in \mathcal{C} \times \mathcal{D}$ such that the sum of digits of cd belongs to a predefined subset of $\mathbb{F}_p$; 3) given a special sequence $\mathcal{Q}$ in $\mathbb{F}_q$, estimate the number of elements of $\mathcal{Q}$ with preassigned digits. For this last problem, we show that we can preassign a positive proportion of digits, in the spirit of a recent result of Bourgain [1] who studied the number of prime numbers with a positive proportion of preassigned digits.

[1] J. Bourgain, *Prescribing the binary digits of primes, II*, Israel J. Math., 206 (2015), pp. 165–182.

[2] C. Dartyge and A. Sárközy, *The sum of digits function in finite fields*, Proc. Amer. Math. Soc., 141 (2013), pp. 4119–4124.