On double correlations of multiplicative functions

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Let $g : \mathbb{N} \to \mathbb{C}$ be a bounded multiplicative function. We consider the logarithmically weighted correlation

$$\frac{1}{\log x} \sum_{n \leq x} \frac{g(n)g(n+1)}{n}. \tag{1}$$

In a recent breakthrough work, Tao showed that if g is non-pretentious, then the correlation (**??**) is $o(1)$. Making a different assumption on g, namely that g has "density" δ (over arithmetic progressions in a suitable sense), we show that the correlation (**??**) has the anticipated value $\delta^2 + o(1)$.

We present applications to consecutive smooth numbers, in particular establishing a logarithmic version of a conjecture of Erdős and Pomerance.