Power monoids of Dedekind-finite, aperiodic monoids are fully elastic

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Let $\mathcal{P}_{\text{fin},0}(\mathbf{N})$ be the collection of all finite subsets of $\mathbf{N}$ containing 0, which we turn into a commutative monoid by endowing it with the operation of set addition

$$(X, Y) \mapsto X + Y := \{x + y : (x, y) \in X \times Y\}.$$

We say that a set $A \in \mathcal{P}_{\text{fin},0}(\mathbf{N})$ is an *atom* if there do not exist $X, Y \in \mathcal{P}_{\text{fin},0}(\mathbf{N})$ with $A = X + Y$ and $|X|, |Y| \geq 2$. Then, given $X \in \mathcal{P}_{\text{fin},0}(\mathbf{N})$, we denote by $\mathsf{L}(X)$ the set of all integers $k \geq 1$ such that $X = A_1 + \cdots + A_k$ for some atoms $A_1, \ldots, A_k \in \mathcal{P}_{\text{fin},0}(\mathbf{N})$.

We establish that (i) $\mathsf{L}(X) \neq \emptyset$ for all $X \in \mathcal{P}_{\text{fin},0}(\mathbf{N})$ with $X \neq \{0\}$, and (ii) for every $m, n \in \mathbf{N}^+$ there is a set $X \in \mathcal{P}_{\text{fin},0}(\mathbf{N})$ with $\mathsf{L}(X) = \{m, n\}$.

It follows, in particular, that $\mathcal{P}_{\text{fin},0}(\mathbf{N})$ is *fully elastic*, and we show how to transfer this conclusion to the power monoid and restricted power monoid of a Dedekind-finite, aperiodic monoid.

The talk is based on joint work with Yushuang Fan (arXiv:1701.09152).