Quantitative Results on Diophantine Equations in Many Variables

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A classical article by Birch [Bir62] implies that a system of polynomials $f_1, \ldots, f_R \in \mathbb{Z}[x_1, \ldots, x_n]$ of the same degree satisfies the smooth Hasse principle if the number of variables n is large compared to the Birch' singular locus. We discuss a quantitative version of this work, yielding asymptotics (in terms of the coefficient of maximal modulus of these polynomials) for the number of integer zeros of this system within a growing box. An application is quantitative strong approximation: assuming the existence of local zeros, we give an upper bound on the smallest non-trivial integer zero provided the affine or projective variety corresponding to the system of polynomials is non-singular.

[Bir62] B. J. Birch. Forms in many variables. *Proc. Roy. Soc. Ser. A*, 265:245–263,1961/1962.