On the classical and the weighted Harborth constant of certain abelian groups

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Let $(G, +, 0)$ be a finite abelian group. The Harborth constant of G, denoted $g(G)$, is the smallest integer t such that every subset A of G of size $|A| \geq t$ contains a subset of size $\exp(G)$ whose elements have sum 0. This constant was introduced by Harborth [H]; it is a variant of the Erdös–Ginzburg–Ziv constant. Its value is so far only known for a few types of groups. For groups of exponent 2 it is easy to see that $g(G) = |G| + 1$ as the sum of two distinct elements is never 0, and for cyclic groups one finds easily that the Harborth constant is equal to $|G|$ if $|G|$ is odd and $|G| + 1$ otherwise.

These simple cases apart the problem becomes challenging. For groups of exponent 3 it is equivalent to the cap-set problem in ternary spaces, a well-known hard problem in discrete geometry and additive combinatorics. For the direct sum of two cyclic groups of prime order $p \geq 67$, it was shown by Gao and Thangadurai [GT] that $g(C_p^2) = 2p - 1$. Moreover, for groups of the form $C_2 \oplus C_{2n}$ it is known by a result of Marchan, Ordaz, Ramos, and Schmid [MORS] that $g(C_2 \oplus C_{2n})$ is equal to $2n + 2$ for even n and equal to $2n + 3$ for odd n.

The talk gives an overview on ongoing joint work with Marchan, Guillot, Ordaz, and Schmid on the value of the Harborth constant for certain groups of exponent 4 as well as groups of the form $C_2^2 \oplus C_{2n}$ and $C_3 \oplus C_{3n}$.

Time permitting, the analogue constants with weights (see for example [MORS2]) will also be discussed.

[GT] W.D. Gao and R. Thangadurai. A variant of Kemnitz conjecture. *Journal of Combinatorial Theory, Series A* 107.1 (2004): 69–86.

[H] H. Harborth. Ein Extremalproblem für Gitterpunkte. *Journal für die reine und angewandte Mathematik* 262 (1973): 356–360.

[MORS] L.E. Marchan, O. Ordaz, D. Ramos, W. A. Schmid. Some exact values of the Harborth constant and its plus-minus weighted analogue. *Archiv der Mathematik* 101 (2013), 501–512.

[MORS2] L.E. Marchan, O. Ordaz, D. Ramos, W. A. Schmid. Inverse results for weighted Harborth constants. *International Journal of Number Theory* 12.07 (2016): 1845–1861.