The set of minimal distances and the characterization of class groups in Krull monoids

Qinghai Zhong
University of Graz

Let H be a Krull monoid with finite class group G such that every class contains a prime divisor. Then every non-unit $a \in H$ can be written as a finite product of atoms, say $a = u_1 \cdot \ldots \cdot u_k$. The set $\mathsf{L}(a)$ of all possible factorization lengths k is called the set of lengths of a. There is a constant $M \in \mathbb{N}$ such that all sets of lengths are almost arithmetical multiprogressions with bound M and with difference $d \in \Delta^*(H)$, where $\Delta^*(H)$ denotes the set of minimal distances of H. We study the structure of $\Delta^*(H)$ and characterize the class group for which $\Delta^*(H)$ is an interval.

It is classical that the system $\mathcal{L}(H) = \{\mathsf{L}(a) \mid a \in H\}$ of all sets of lengths depends only on the class group G, and a standing conjecture states that conversely the system $\mathcal{L}(H)$ is characteristic for the class group. We verify the conjecture if the class group is isomorphic to C_n^r with $r, n \in \mathbb{N}$ and $\Delta^*(H)$ is not an interval.