Berkovich spaces: 30 years

Titles and abstracts

Karim Adiprasito (Hebrew University of Jerusalem): *Log smoothness and polystability over valuation rings*

Let O be a valuation ring of height one of residual characteristic exponent p and with algebraically closed field of fractions. Our main result provides a best possible resolution of the monoidal structure M_X of a log variety X over O: there exists a log modification $Y \to X$ such that the monoidal structure of Y is polystable. In particular, if X is log smooth over O then Y is polystable. As a corollary we deduce that any log variety over O possesses a polystable alteration of degree p^n. The core of our proof is a subdivision result for polyhedral complexes satisfying certain rationality conditions.

Joint work with G. Liu, I. Pak, and M. Temkin.

Matt Baker (Georgia Institute of Technology): *Hyperfields, Ordered Blueprints, and Moduli Spaces of Matroids*

In tropical geometry, linear spaces are in 1-1 correspondence with "valuated matroids". I will discuss a unified theory which views linear subspaces, ordinary matroids, and valuated matroids as special cases of "matroids over hyperfields". I will then discuss the geometric construction of a "universal Grassmannian" which, via base change, yields (fine) moduli spaces for matroids over hyperfields. The universal Grassmannian is constructed using Oliver Lorscheid's theory of ordered blueprints, which is also useful for understanding the relationship between Berkovich spaces, skeletons, and tropicalizations.

This is joint work with Nathan Bowler and Oliver Lorscheid.

Francesco Baldassarri (University of Padova): *Entire Fourier expansions on* $\mathbf{Q}_p$

We consider the $\mathbf{Q}_p$-algebra $\mathscr{E}$ of power series in $\mathbf{Q}_p[\![x]\!]$ which represent p-adically entire functions on $\mathbf{C}_p$ that restrict to bounded uniformly continuous maps $\mathbf{Q}_p \to \mathbf{Q}_p$. Then $\mathscr{E}$ is a $\mathbf{Q}_p$-Fréchet for the topology of uniform convergence on bounded subsets of $\mathbf{C}_p$ and on $\mathbf{Q}_p$. We exhibit a topological basis $\{G_q\}_{q\in S}$ of $\mathscr{E}$, where $S = \mathbf{Z}[1/p] \cap \mathbf{R}_{\geq 0}$, such that

$$G_q(x+y) = \sum_{q_1+q_2=q} G_{q_1}(x)G_{q_2}(y),\ \forall q \in S,$$

where the net of finite partial sums converges uniformly on bounded subsets of $\mathbf{C}_p^2$. We suitably identify the algebra of functions on the formal perfectoid open unit disc $\mathbf{D}$ over $\mathbf{Z}_p$, namely the (p,T)-adic completion $\mathscr{D}$ of the ring $\mathbf{Z}_{(p)}[T^{1/p^\infty}]$, with the topological Hopf algebra of $\mathbf{Z}_p$–valued uniform measures on $\mathbf{Q}_p$ and describe a special element $\mu_{\mathrm{can}} \in \mathscr{D}$, called the

canonical measure. For any $q \in S$, the measure μ_{can}^q is well-defined in $\mathscr{D}$ and any bounded uniformly continuous function $f\colon \mathbf{Q}_p \to \mathbf{Q}_p$ admits the generalized Amice-Fourier expansion

$$f = \sum_{q\in S} \left(\int_{\mathbf{Q}_p} f \mu_{\mathrm{can}}^{\mu} \right) = \text{“} \sum_{q\in S} f^{[q]}(0) G_q \text{"}$$

where the net of finite partial sums converges uniformly on compact subsets of $\mathbf{Q}_p$. The tautological formula

$$\mu = \sum_{q\in S} \left(\int_{\mathbf{Q}_p} G_q(x)\mu(x) \right) \mu_{\mathrm{can}}^q$$

for any uniform measure μ, may be regarded as a generalized Amice transform which identifies μ with a perfectoid function on $\mathbf{D}$.

We will clarify our construction by calculating the untilted form of the Artin-Hasse isomorphism of perfectoid discs in characteristic $p > 0$.

Antoine Chambert–Loir (Université Paris-Diderot): *A non-archimedean Ax-Lindemann theorem*

A significant step in the Pila-Zannier approach to the André-Oort conjecture is a geometric transcendence result for the uniformization map of modular curves. I will discuss joint work with François Loeser. We prove an analogue of this result in non-archimedean geometry, namely for the uniformization of Mumford curves whose associated fundamental groups are non-abelian Schottky subgroups of $\mathrm{PGL}(2, \bar{\mathbf{Q}}_p)$ contained in $\mathrm{PGL}(2, \bar{\mathbf{Q}})$. In particular, we characterize bi-algebraic irreducible subvarieties of the uniformization.

Hélène Esnault (Freie Universität Berlin): *Cohomological dimension in pro-p-towers*

We give a proof without use of perfectoid geometry of Scholze's vanishing theorem beyond the dimension of cohomology with $\mathbf{F}_p$-coefficients of projective varieties in a specific pro-p-tower.

Charles Favre (École Polytechnique): *Hybrid spaces and dynamics*

We shall review how the construction of V. Berkovich of hybrid spaces can be used to get a control on the blow-up of Lyapunov exponents in degenerating families of analytic dynamical systems.

Jean-Marc Fontaine (Université Paris-Sud): *Almost C_p-representations and vector bundles*

Let K be a finite extension of $\mathbf{Q}_p$ and G_K its absolute Galois group. Long time ago, I introduced the abelian category of almost C_p-representations which contains the category of p-adic representations of G_K. The group G_K acts on the fundamental curve X of p-adic Hodge theory. I proved with Laurent Fargues that there is an equivalence between the category of G_K-equivariant vector bundles on X which are semi-stable of slope 0 and p-adic representations of G_K. I will explain that this equivalence can be extended to a construction with which one can recover the category of almost C_p-representations of G_K from the category of G_K-equivariant $\mathcal{O}_X$-modules and conversely.

Walter Gubler (Universität Regensburg): *Equidistribution measures and delta-forms on Berkovich spaces*

Chambert–Loir introduced measures on Berkovich spaces which arise naturally for equidistribution results. We will explain some related results. Then we will show how the formalism of delta-forms from a joint paper with Klaus Kuennemann can be used to write these measures as a wedge product of first Chern forms. As an additional tool to the real forms of Chambert–Loir and Ducros, we use stable tropical intersection theory. As an application, we will give a non-archimedean Arakelov theory for metrized divisors.

Luc Illusie (Université Paris-Sud): *A new approach to the de Rham-Witt theory, after Bhatt, Lurie and Mathew*

Bhatt, Lurie, and Mathew have recently constructed de Rham-Witt-like complexes for schemes over a perfect field of positive characteristic, which coincide with the classical one in the smooth case and are of interest in certain singular cases. I will explain their approach, which exploits the (fashionable) Deligne-Ogus décalage η_p functor, and uses only elementary homological algebra (in particular, avoids the laborious calculations involved in the so-called canonical bases for Deligne complexes of integral forms).

Mattias Jonsson (University of Michigan): *Berkovich spaces and K-stability*

K-stability is a property of a polarized complex variety that is conjecturally equivalent to the existence of special Kähler metrics in the cohomology class of the polarization—a conjecture that was recently settled in the case of Fano manifolds with the anticanonical polarization. I will describe joint work with Boucksom and Hisamoto, where we study K-stability using semipositive metrics on the Berkovich analytification on the polarization, with respect to the trivial absolute value on complex numbers.

François Loeser (Sorbonne Université): *When non-archimedean geometry meets non-standard analysis*

I will report on joint work with Antoine Ducros and Ehud Hrushovski connecting the Chambert–Loir and Ducros theory of real differential forms on Berkovich spaces to the more classical complex theory. This is performed by working over some non-standard model of the field of complex numbers, which is endowed at the same time with an archimedean and a non-archimedean norm. In this context we show that non-archimedean forms and integrals considered by Chambert–Loir and Ducros arise naturally as limits of their archimedean counterparts.

Florent Martin (Universität Regensburg): *Differentiability of non-Archimedean volumes*

If L is a line bundle on a projective variety Y of dimension n, the *volume* of L is defined as

$$\mathrm{vol}_Y(L) := \lim_{m\in\mathbb{N}} \frac{h^0(L^{\otimes m})}{m^n/n!}.$$

The volume can be continuously extended to the real Néron-Severi group of Y (a real vector space) and Boucksom, Favre and Jonsson proved that $\mathrm{vol}_Y(\cdot)$ is even C^1. If Y is additionally a complex analytic variety, and L is equipped with a metric $\|\cdot\|$, Boucksom and Berman have

proved similar properties for the asymptotic growth of the set of global sections s of $L^{\otimes m}$ with $\|s\| \leq 1$.

I will explain analogous differentiability results when Y is an analytic variety over a discretely valued non-Archimedean field, thus proving a formula conjectured by Kontsevich and Tschinkel. The proof is strongly inspired by ideas of Yuan, and relies on holomorphic Morse inequalities. The main motivation of this work is to drop some hypotheses in some results of Boucksom, Favre and Jonsson about non-Archimedean Monge-Ampère equations.

This is a joint work with J. Burgos, W. Gubler, P. Jell and K. Künnemann.

Johannes Nicaise (Imperial College London): *The non-archimedean SYZ fibration*

This talk is based on joint work with Chenyang Xu and Tony Yue Yu. I will explain the construction of the non-archimedean Strominger-Yau-Zaslow fibration, whose existence was conjectured by Kontsevich and Soibelman in their non-archimedean approach to Mirror Symmetry. I will also explain why it is an affinoid torus fibration away from a codimension two subset of the base, as predicted by Kontsevich and Soibelman. The proof is based on the Minimal Model Program in birational geometry.

Sam Payne (Yale University): *Tropical curves, graph complexes, and top weight cohomology of M_g*

I will discuss the topology of a space of stable tropical curves of genus g with volume 1, naturally identified with the link of the vertex in the skeleton of M_g^{an} associated to the Deligne-Mumford compactification. It follows from Berkovich's analogue of the Tate Conjecture that the reduced rational homology of this space is canonically identified with the top weight cohomology of M_g, and we show that it is also identified with the homology of Kontsevich's graph complex. As one application, we show that $\mathrm{H}^{4g-6}(M_g)$ is nonzero for infinitely many g. This disproves a recent conjecture of Church, Farb, and Putman as well as an older, more general conjecture of Kontsevich.

Joint work with M. Chan and S. Galatius.

Amaury Thuillier (Université de Lyon 1): *On the homotopy type of analytifications of algebraic varieties*

Some years ago, Hrushovski and Loeser developped an analogue of Berkovich geometry into the framework of the model theory of valued fields, and deduced from it many results on homotopy types of analytification of quasi-projective varieties over a non-archimedean field. I will describe an alternative approach based on alterations and Berkovich's construction of skeleta and homotopies.

Annette Werner (Goethe-Universität Frankfurt): *Non-archimedean and tropical Hodge bundle*

We study the image of the tropicalization map connecting the Berkovich analytic and the tropical Hodge bundle. For every pair consisting of a stable tropical curve G plus a divisor in the canonical linear system on G, we obtain a combinatorial condition to decide whether there is a smooth curve over a non-Archimedean field whose stable reduction has G as its dual graph together with an effective canonical divisor specializing to the given one. This is joint work with Martin M?ller and Martin Ulirsch.

Tony Yue Yu (Université Paris-Sud): *Enumeration of non-archimedean curves in higher dimensional log Calabi-Yau varieties*

I will discuss the enumeration of non-archimedean curves in higher dimensional affine log Calabi-Yau varieties containing an open algebraic torus, part of my joint work with S. Keel. This generalizes the previously studied two-dimensional case, and includes cluster varieties arising from representation theory. Many new ideas are developed in order to go beyond the two-dimensional case. In my talk, I will explain various properties of the moduli spaces which lead to the enumeration. Moreover, I will introduce a new notion of "skeletal curves", curves whose skeleton lies in the essential skeleton of the ambient log Calabi-Yau variety. Such curves play a special role in the theory.

Shou-Wu Zhang (Princeton University): *Admissible height pairings of algebraic cycles*

For a smooth and projective variety X over a global field of dimension n with an adelic polarization, we propose some local and global height pairings for two cycles Y, Z of pure dimension p, q satisfying $p + q = n - 1$.

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00 - 10:00	09:30 – 10:30 Illusie	09:30 – 10:30 Chambert–Loir	09:00 – 10:00 Payne	09:30 – 10:30 Zhang	09:00 – 10:00 Baldassarri
10:00 - 11:00	10:30 – 11:00 coffee	10:30 – 11:00 coffee	10:20 – 11:20 Nicaise	10:30 – 11:00 coffee	10:20 – 11:20 Adiprasito
11:00 - 12:00	11:00 – 12:00 Martin	11:00 – 12:00 Favre	11:20 – 11:50 coffee	11:00 – 12:00 Fontaine	11:20 – 11:50 coffee
12:00 - 13:00			11:50 – 12:50 Baker		11:50 – 12:50 Yu
13:00 - 14:00					
14:00 - 15:00	14:00 – 15:00 Werner	14:00 – 15:00 Gubler		14:00 – 15:00 Esnault	
15:00 - 16:00	15:00 – 15:30 coffee	15:00 – 15:30 coffee		15:00 – 15:30 coffee	
16:00 - 17:00	15:30 – 16:30 Thuillier	15:30 – 16:30 Jonsson		15:30 – 16:30 Loeser	
		16:30 – 17:00 Poster session			